QUIZ 26: LESSON 33
APRIL 17, 2019

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & 3 & -2 \\ 1 & 2 & -1 \\ -2 & -1 & 1 \end{bmatrix}$$

Solution: Write

$$[A|I] = \left[\begin{array}{ccc|ccc} 2 & 3 & -2 & 1 & 0 & 0 \\ 1 & 2 & -1 & 0 & 1 & 0 \\ -2 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1+R_3\to R_3} \left[\begin{array}{ccc|ccc} 2 & 3 & -2 & 1 & 0 & 0 \\ 1 & 2 & -1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{-R_2+R_1\to R_1} \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & -1 & 0 \\ 1 & 2 & -1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 1 & 0 & 1 \end{array}\right] \xrightarrow{-R_1+R_2\to R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{-2R_2+R_3\to R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 3 & -4 & 1 \end{array}\right] \xrightarrow{-R_3\to R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 \\ 0 & 0 & 1 & -3 & 4 & -1 \end{array}\right]$$

$$\xrightarrow{R_3+R_1\to R_1} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & -1 & 2 & 0 \\ 0 & 0 & 1 & -3 & 4 & -1 \end{array}\right] \xrightarrow{-R_2+R_1\to R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 1 & 0 & -1 & 2 & 0 \\ 0 & 0 & 1 & -3 & 4 & -1 \end{array}\right]$$

Hence,

$$A^{-1} = \begin{bmatrix} -1 & 1 & -1 \\ -1 & 2 & 0 \\ -3 & 4 & -1 \end{bmatrix}$$

2. [5 pts] Solve the following system of equations:

$$\begin{cases} 2x + 3y - 2z = -4 \\ x + 2y - z = -3 \\ -2x - y + z = -1 \end{cases}$$

HINT: Use the inverse matrix that you found for # 1.

Solution: Let

$$A = \begin{bmatrix} 2 & 3 & -2 \\ 1 & 2 & -1 \\ -2 & -1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad Y = \begin{bmatrix} -4 \\ -3 \\ -1 \end{bmatrix}.$$

Then this system of equations is given by

$$AX = Y$$

$$\begin{bmatrix} 2 & 3 & -2 \\ 1 & 2 & -1 \\ -2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ -3 \\ -1 \end{bmatrix}$$

Thus, $X = A^{-1}Y$ which implies

$$\begin{aligned} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \begin{bmatrix} -1 & 1 & -1 \\ -1 & 2 & 0 \\ -3 & 4 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ -3 \\ -1 \end{bmatrix} \\ &= \begin{bmatrix} -1(-4) + 1(-3) + (-1)(-1) \\ -1(-4) + 2(-3) + 0(-1) \\ -3(-4) + 4(-3) + (-1)(-1) \end{bmatrix} \\ &= \begin{bmatrix} 4 - 3 + 1 \\ 4 - 6 + 0 \\ 12 - 12 + 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \end{aligned}$$